Stability Analysis of Passive Dynamic Walking for Bipedal Robots

Christian Hubicki[1], Professor Keith Buffinton[1], C. David Remy[2]

Department of Mechanical Engineering[1]
Bucknell University, Lewisburg, PA

Swiss Federal Institute of Technology[2]
Zurich, Switzerland

Abstract

Bipedal walking has long been a goal in the field of mobile robotics. While prototype robots have been built that use two-legged locomotion, they have yet to be put into practical use. This lack of widespread implementation is due in no small part to inherent energy inefficiency of the specific walking gait being used. Development of a more efficient gait is essential to bipedal robots in most real-world applications.

Another approach for determining an appropriate gait trajectory is to mimic the effect of gravity to help achieve a more efficient stride. Humans typically walk with their centers of gravity ahead of their planted foot, allowing gravity to contribute to their forward motion. A simple method of modeling this phenomenon is to place an unpowered biped on a slight incline. When such a model is evaluated, it has been found that an un-powered bipedal walker can take steps perpetually down the incline. This resulting “passive” walker has given rise to the field of *passive dynamic walking*.

The focus of this research is to identify sets of parameters which satisfy the conditions for limit-cycle walking. Furthermore, computation of Floquet multipliers for such limit-cycle conditions allows for stability analysis of proposed gait trajectories. This method of evaluating the bipedal system dynamics can assess sensitivity to initial conditions for any set of parameters. The results of this numerical analysis can be used as a theoretical foundation for developing fully-actuated gait trajectories and produce more efficient walking in bipedal robots.

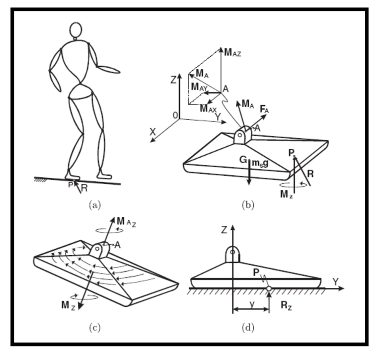
Zero-Moment Point vs. Passive-Dynamic Based Methods

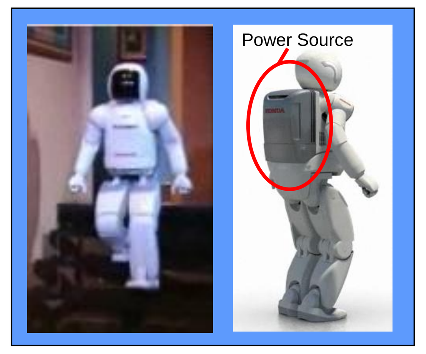
Zero-Moment Point

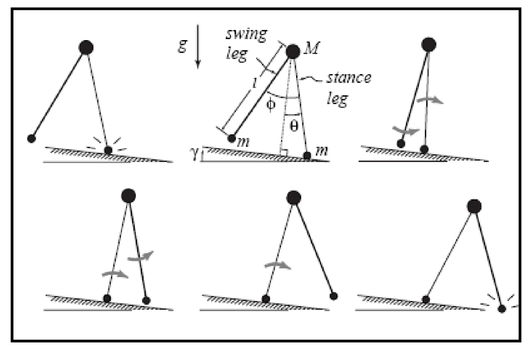
- Robot is balanced at all times over the center of pressure of the foot
- Easier to control and very “stable”
- Very energy inefficient

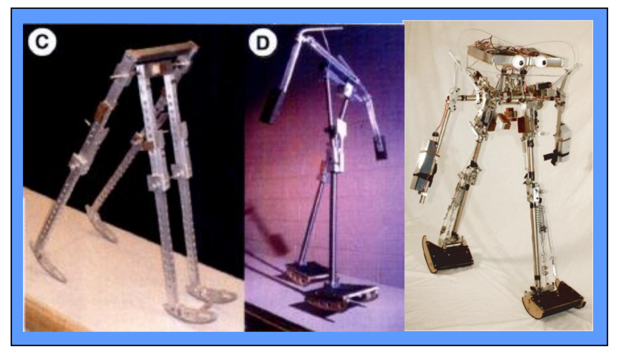
Passive-Dynamic Based

- Robot is generally “falling forward”, using gravity to its advantage
- Very energy efficient
- Difficult to keep stable and robust



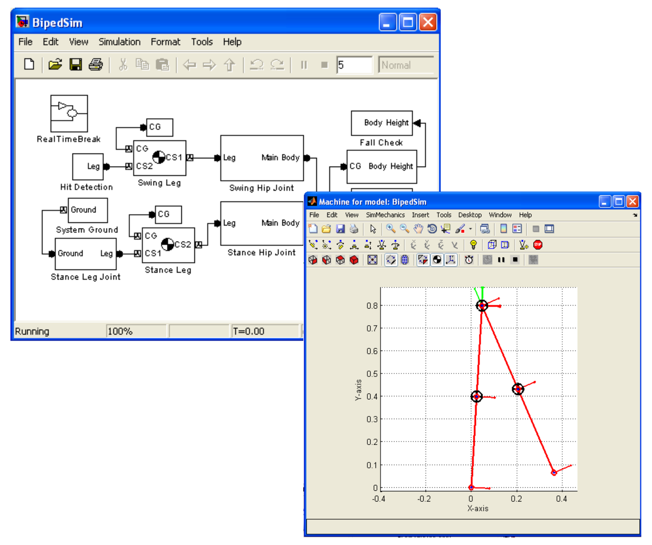






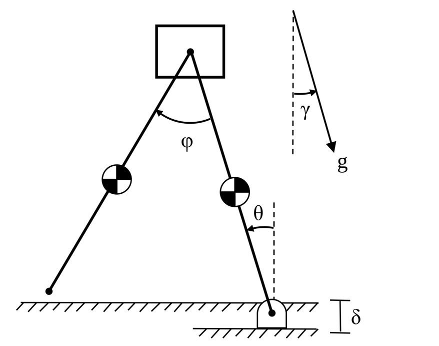
Simulation

SimMechanics Model and Visualization

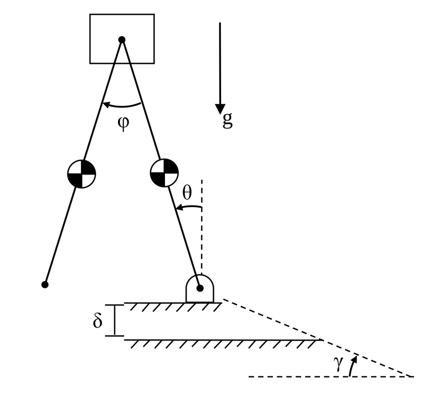


Basic Terrain Modeling

Uneven Terrain Model



Shallow Step Model

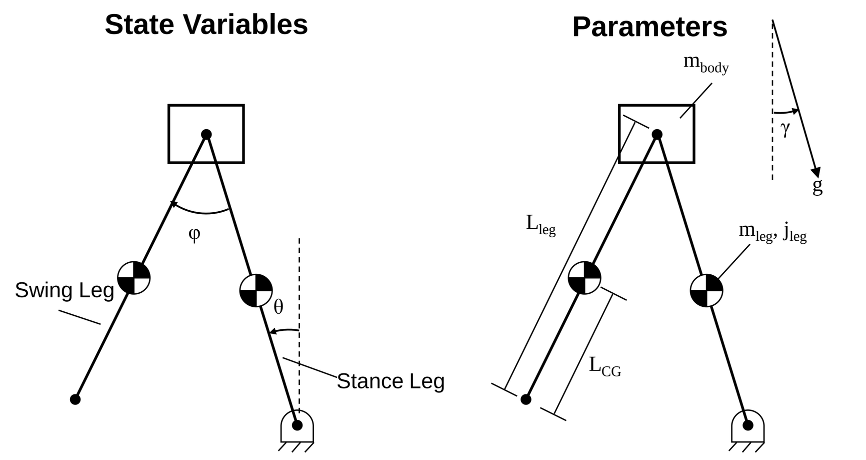


Stability Analysis Method

Parameter Study:

1. Set model parameters
2. Using a numerical solver, identify initial conditions which yield limit-cycle walking $\quad P(\vec{x}^*) - \vec{x}^* = 0$
3. Perturb initial conditions by a “small” amount and observe its effect on the gait $\quad \vec{x}^{*+1} = P(\vec{x}^*) + \Delta\vec{x}$
4. Compute eigenvalues (Floquet multipliers) to evaluate stability $\quad J = \left.\frac{\partial P(\vec{x})}{\partial \vec{x}}\right|_{\vec{x}^*}$
5. Iterate parameters and repeat

Biped Simulation Model

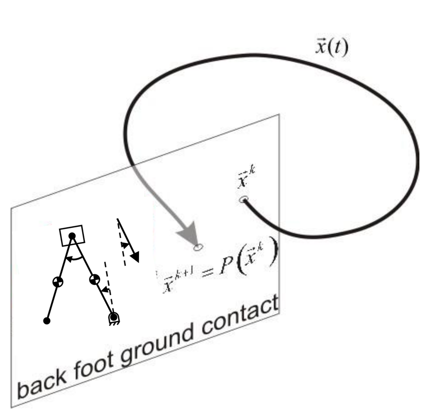


A simple passive walker is modeled as a modified double-pendulum. An inclined surface is modeled by adjusting the direction of gravity.

Poincare Sections and Floquet Multipliers

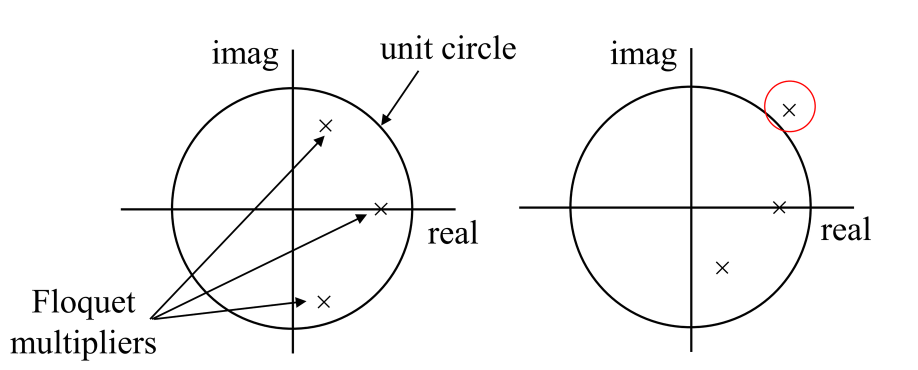
The goal of this analysis is to determine whether the current conditions result in both a periodic and stable gait. A *Poincare section*, P(·), is defined to be the instance when the back foot collides with the ground. States which map onto themselves via the stride-to-stride transfer function are periodic and are considered to be in limit cycle conditions.

$$\vec{x}^{k+1} = P(\vec{x}^k)$$



Once limit cycle conditions are identified, a small disturbance, Δx, is applied to the initial conditions. By running the simulation with these perturbed conditions, a linearized Jacobian of the transfer function is calculated. The eigenvalues of the Jacobian, *Floquet multipliers*, determine the stability of the limit cycle. All eigenvalues must lie within the unit circle in order to be considered stable.

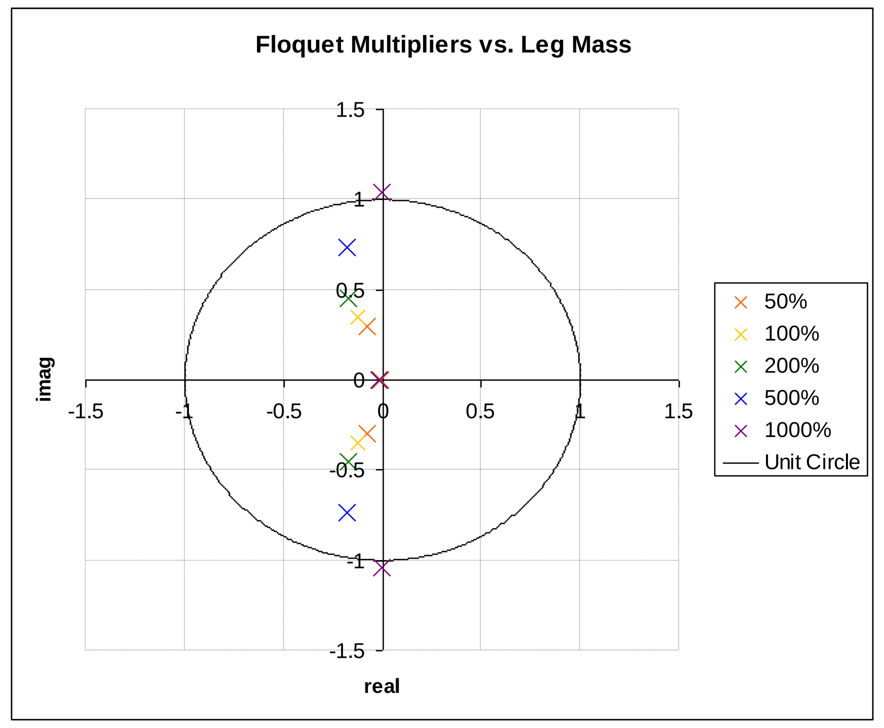
$$\vec{x}^{*+1} = P(\vec{x}^*) + \Delta\vec{x}$$

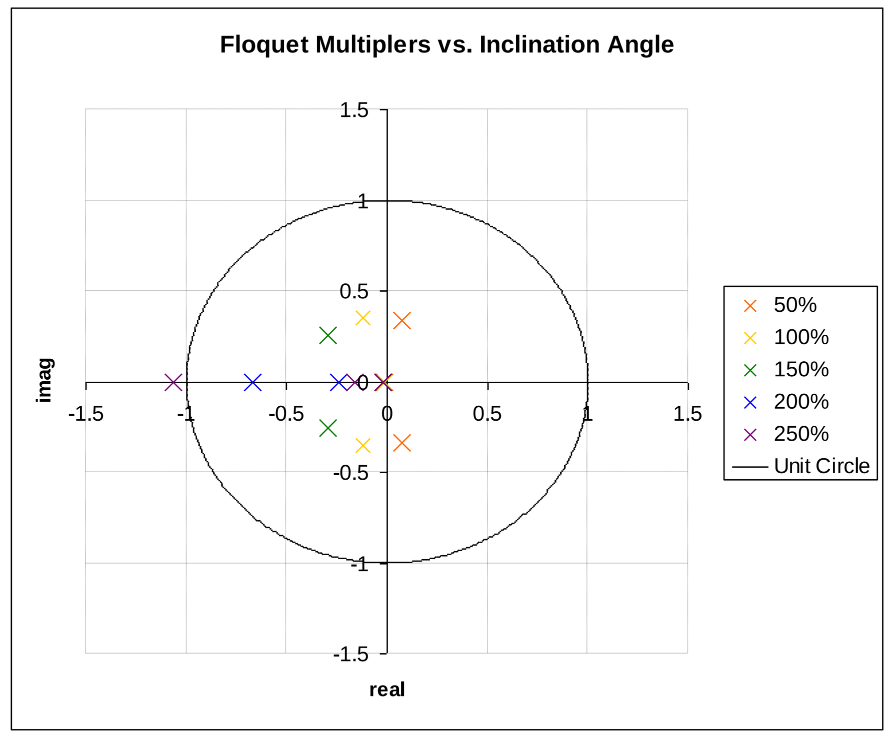


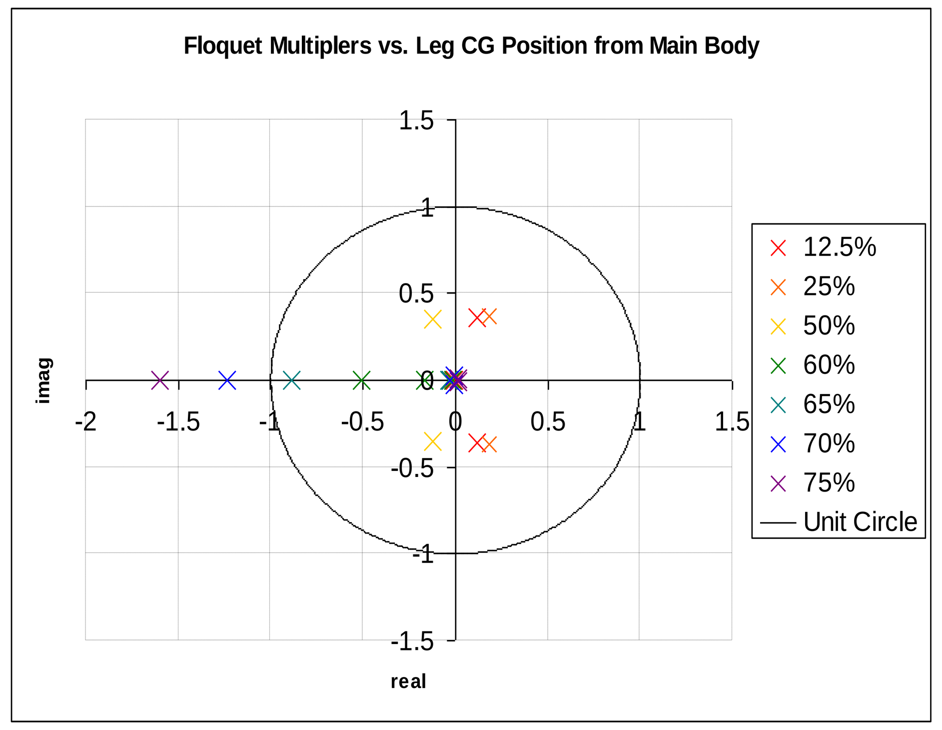
Stable **Unstable**

Parameter Study Results

From this numerical experiment, the following parameter changes tend to make the system unstable: increasing the mass distribution to the legs, shifting the CG away from the main body, and increasing the incline on which the biped walks.







Conclusion and Future Work

This numerical study has shown that passive-dynamic bipedal walkers have a relatively wide range of parameters which yield a stable, periodic gait. Furthermore, these results indicate how parameters should be altered in order to achieve stability in an otherwise unstable system.

The next stage of research involves terrain modeling. For a bipedal robot to be effective in most real-world applications, the walker should be able to maintain its gait when varying the geometry of the ground beneath it. Shown above are suggested models for assessing the effects of terrain on a bipedal walker. Further work should include the addition of actuation and removing the incline which allows the robot to walk. This is a necessary step towards satisfying applications as most real-world situations do not involve a steady, downward incline.

References

1. McGeer, T., 1990, “Passive Dynamic Walking,” *Int. J. Robotics Res.*, **9**, 62-82.
2. Kuo, A., 2002, “Energetics of Actively Powered Locomotion Using the Simplest Walking Model,” *J. Biomech. Eng.*, **124**, 113-120.

Acknowledgements

The authors gratefully acknowledge support for this work provided by the Office of Naval Research under Grant No. N000140810953.